

# Reconstructing interacting new agegraphic polytropic gas model in non-flat FRW universe

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## Abstract

We study the correspondence between the interacting new agegraphic dark energy and the polytropic gas model of dark energy in the non-flat FRW universe. This correspondence allows to reconstruct the potential and the dynamics for the scalar field of the polytropic model, which describe accelerated expansion of the universe.

**Keywords:** Dark energy theory – Polytropic model

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# 1 Introduction

Type Ia supernovae observational data suggest that the universe is dominated by two dark components containing dark matter and dark energy (Riess et al. 1998; Perlmutter et al. 1999; de Bernardis et al. 2000; Perlmutter et al. 2003). Dark matter (DM), a matter without pressure, is mainly used to explain galactic curves and large-scale structure formation, while dark energy (DE), an exotic energy with negative pressure, is used to explain the present cosmic accelerating expansion. However, the nature of DE is still unknown, and people have proposed some candidates to describe it (for a good review see Karami et al. 2009a).

Recently, the original agegraphic dark energy (OADE) and new agegraphic dark energy (NADE) models were proposed by Cai (2007) and Wei and Cai (2008a), respectively. Cai (2007) proposed the OADE model to explain the accelerated expansion of the universe, based on the uncertainty relation of quantum mechanics as well as the gravitational effect in general relativity. The OADE model had some difficulties. In particular, it cannot justify the matter-dominated era (Cai 2007). This motivated Wei and Cai (2008a) to propose the NADE model, while the time scale is chosen to be the conformal time instead of the age of the universe. The evolution behavior of the NADE is very different from that of the OADE. Instead the evolution behavior of the NADE is similar to that of the holographic DE (Cohen et al. 1999; Horava and Minic 2000; Thomas 2002; Li 2004; Zhang and Wu 2005; Zhang 2006; Zhang and Wu 2007; Li et al. 2009a,b; Sheykhi 2009b; Gao et al. 2009; Karami 2010). But some essential differences exist between them. In particular, the NADE model is free of the drawback concerning causality problem which exists in the holographic DE model. The ADE models assume that the observed DE comes from the spacetime and matter field fluctuations in the universe (Wei and Cai 2008a, 2009). The ADE models have been studied in ample detail by Kim et al. (2008a), Kim et al. (2008b), Wu et al. (2008), Zhang et al. (2008), Wei and Cai (2008b), Neupane (2009), and Sheykhi (2009a, 2010b).

Karami et al. (2009a) introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. An example of a polytropic gas is a gas where the pressure is dominated by degenerate electrons in white dwarfs or degenerate neutrons in neutron stars. Another example is the case where pressure and density are related adiabatically in main sequence stars (Karami et al. 2009a).

Reconstructing the holographic and agegraphic scalar field models of DE is one of interesting issue which has been investigated in the literature. For instance, holographic quintom (Zhang 2006), holographic quintessence (Zhang 2007), holographic tachyon (Zhang et al. 2007), holographic Ricci quintom (Zhang 2009), new holographic quintessence, tachyon, K-essence and dilaton (Granda and Oliveros 2009; Karami and Fehri 2010), interacting new agegraphic tachyon, K-essence and dilaton (Karami et al. 2009b), and interacting agegraphic tachyon (Sheykhi 2010a).

All mentioned in above motivate us to investigate the correspondence between the interacting NADE and the polytropic gas model of DE in the non-flat FRW universe. This paper is organized as follows. In Section 2, we study the interacting NADE with the cold DM in the non-flat FRW universe. In Section 3, we investigate the polytropic gas model of DE. In Section 4, we suggest a correspondence between the interacting NADE

and the polytropic gas model of DE. We reconstruct the potential and the dynamics for the scalar field of the polytropic model, which describe accelerated expansion. Section 5 is devoted to conclusions.

## 2 Interacting NADE model in non-flat FRW universe

We consider the Friedmann-Robertson-Walker (FRW) metric for the non-flat universe as

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right), \quad (1)$$

where  $a$  is the cosmic scale factor and  $k = 0, 1, -1$  represent a flat, closed and open FRW universe, respectively. Observational evidences support the existence of a closed universe with a small positive curvature ( $\Omega_k \sim 0.02$ ) (Bennett et al. 2003; Spergel 2003; Tegmark et al. 2004; Seljak et al. 2006; Spergel et al. 2007). Besides, as usually believed, an early inflation era leads to a flat universe. This is not a necessary consequence if the number of e-foldings is not very large (Huang and Li 2004). It is still possible that there is a contribution to the Friedmann equation from the spatial curvature when studying the late universe, though much smaller than other energy components according to observations.

For the non-flat FRW universe containing the DE and DM, the first Friedmann equation has the following form

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2} (\rho_\Lambda + \rho_m), \quad (2)$$

where  $\rho_\Lambda$  and  $\rho_m$  are the energy density of DE and DM, respectively. Let us define the dimensionless energy densities as

$$\begin{aligned} \Omega_m &= \frac{\rho_m}{\rho_{\text{cr}}} = \frac{\rho_m}{3M_p^2 H^2}, \\ \Omega_\Lambda &= \frac{\rho_\Lambda}{\rho_{\text{cr}}} = \frac{\rho_\Lambda}{3M_p^2 H^2}, \\ \Omega_k &= \frac{k}{a^2 H^2}, \end{aligned} \quad (3)$$

then, the first Friedmann equation yields

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k. \quad (4)$$

Following Sheykhi (2009a), the energy density of the NADE is given by

$$\rho_\Lambda = \frac{3n^2 M_p^2}{\eta^2}, \quad (5)$$

where the numerical factor  $3n^2$  is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved spacetime (since the energy density is derived for Minkowski spacetime), and so on. The astronomical data

for the NADE gives the best-fit value (with  $1\sigma$  uncertainty)  $n = 2.716_{-0.109}^{+0.111}$  (Wei and Cai 2008b). It was found that the coincidence problem could be solved naturally in the NADE model provided that the single model parameter  $n$  is of order unity (Wei and Cai 2008b). Also  $\eta$  is conformal time of the FRW universe, and given by

$$\eta = \int \frac{dt}{a} = \int_0^a \frac{da}{Ha^2}. \quad (6)$$

Note that in the energy density of the OADE model, the age of the universe is appeared in Eq. (5) instead of  $\eta$ . This causes some difficulties. In particular it fails to describe the matter dominated epoch properly (Cai 2007). The DE density (5) has the same form as the holographic DE, but the conformal time stands instead of the future event horizon distance of the universe. Thus the causality problem in the holographic DE is avoided. Because the existence of the future event horizon requires an eternal accelerated expansion of the universe (Wei and Cai 2008a).

From definition  $\rho_\Lambda = 3M_p^2 H^2 \Omega_\Lambda$ , we get

$$\eta = \frac{n}{H\sqrt{\Omega_\Lambda}}. \quad (7)$$

We consider a universe containing an interacting NADE density  $\rho_\Lambda$  and the cold dark matter (CDM), with  $\omega_m = 0$ . The energy equations for NADE and CDM are

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \quad (8)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (9)$$

where following Kim et al. (2006), we choose  $Q = \Gamma\rho_\Lambda$  as an interaction term and  $\Gamma = 3b^2 H(\frac{1+\Omega_k}{\Omega_\Lambda})$  is the decay rate of the NADE component into CDM with a coupling constant  $b^2$ . Although this expression for the interaction term may look purely phenomenological but different Lagrangians have been proposed in support of it (Tsujikawa and Sami 2004). The choice of the interaction between both components was to get a scaling solution to the coincidence problem such that the universe approaches a stationary stage in which the ratio of DE and DM becomes a constant (Hu and Ling 2006). Note that choosing the  $H$  in the  $Q$ -term is motivated purely by mathematical simplicity. Because from the continuity equations, the interaction term should be proportional to a quantity with units of inverse of time. For the latter the obvious choice is the Hubble factor  $H$ . The dynamics of interacting DE models with different  $Q$ -classes have been studied in ample detail by (Amendola 1999, 2000; Pavon and Zimdahl 2005; Wang et al. 2005; Szydlowski 2006; Tsujikawa 2006; Guo et al. 2007; Caldera-Cabral et al. 2009). It should be emphasized that this phenomenological description has proven viable when contrasted with observations, i.e., SNIa, CMB, large scale structure,  $H(z)$ , and age constraints (Wang et al. 2006, 2007; Feng et al. 2007), and recently in galaxy clusters (Bertolami et al. 2007, 2009; Abdalla et al. 2009).

Taking the time derivative of Eq. (5), using  $\dot{\eta} = 1/a$  and Eq. (7) yields

$$\dot{\rho}_\Lambda = -\frac{2H\sqrt{\Omega_\Lambda}}{na}\rho_\Lambda. \quad (10)$$

Substituting Eq. (10) in (8), gives the equation of state (EoS) parameter of the interacting NADE model as

$$\omega_\Lambda = -1 + \frac{2\sqrt{\Omega_\Lambda}}{3na} - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right), \quad (11)$$

which shows that in the absence of interaction between NADE and CDM,  $b^2 = 0$ ,  $\omega_\Lambda$  is always larger than -1 and cannot cross the phantom divide. However, in the presence of interaction,  $b^2 \neq 0$ , taking  $\Omega_\Lambda = 0.72$ ,  $\Omega_k = 0.02$  (Bennett et al. 2003),  $n = 2.7$  (Wei and Cai 2008b) and  $a = 1$  for the present time, Eq. (11) gives

$$\omega_\Lambda = -0.79 - 1.42b^2, \quad (12)$$

which clears that the phantom EoS  $\omega_\Lambda < -1$  can be obtained when  $b^2 > 0.15$  for the coupling between NADE and CDM.

### 3 The polytropic gas model of DE

Following Karami et al. (2009a), the polytropic gas equation of state (EoS) is given by

$$p_\Lambda = K \rho_\Lambda^{1+\frac{1}{n}}, \quad (13)$$

where  $K$  is a positive constant and  $n$  is the polytropic index. In this model, the energy density evolves as

$$\rho_\Lambda = \left( Ba^{\frac{3}{n}} - K \right)^{-n}, \quad (14)$$

where  $B$  is a positive integration constant.

Using Eqs. (13) and (14), the EoS parameter of the polytropic gas model of DE is obtained as

$$\omega_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = -1 - \frac{Ba^{\frac{3}{n}}}{K - Ba^{\frac{3}{n}}}. \quad (15)$$

We see that for  $K > Ba^{\frac{3}{n}}$ ,  $\omega_\Lambda < -1$ , which corresponds to a universe dominated by phantom dark energy. Note that to have  $\rho_\Lambda > 0$ , from Eq. (14) the polytropic index should be even,  $n = (2, 4, 6, \dots)$ .

Following Copeland et al. (2006), one can obtain a corresponding potential for the polytropic gas by treating it as an ordinary scalar field  $\phi(t)$ . Using Eqs. (13), (14) together with  $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$  and  $p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ , we find

$$\dot{\phi}^2 = \frac{Ba^{\frac{3}{n}}}{\left( Ba^{\frac{3}{n}} - K \right)^{n+1}}, \quad (16)$$

$$V(\phi) = \frac{\frac{B}{2} a^{\frac{3}{n}} - K}{\left( Ba^{\frac{3}{n}} - K \right)^{n+1}}. \quad (17)$$

Equation (16) shows that for  $K > Ba^{\frac{3}{n}}$ ,  $\dot{\phi}^2 < 0$ . Therefore one can conclude that the scalar field  $\phi$  is a phantom field. Therefore, a phantom-like equation of state can be generated from the polytropic gas DE model in a non-flat universe.

## 4 Correspondence between the interacting NADE and polytropic gas model of DE

Here we suggest a correspondence between the interacting NADE model with the polytropic gas model of DE in the non-flat universe. To establish this correspondence, we compare the NADE density (5) with the corresponding polytropic gas model density (14) and also equate the EoS parameter of the interacting NADE (11) with the EoS parameter given by (15).

Equating Eqs. (5), (14) and using (7) we obtain

$$K = Ba^{\frac{3}{n}} - (3M_P^2 H^2 \Omega_\Lambda)^{\frac{-1}{n}}. \quad (18)$$

Equating Eqs. (11), (15) and using (18), we get

$$K = (3M_P^2 H^2 \Omega_\Lambda)^{\frac{-1}{n}} \left[ -1 + \frac{2\sqrt{\Omega_\Lambda}}{3na} - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) \right]. \quad (19)$$

Substituting Eq. (19) in (18) reduces to

$$B = (3M_P^2 H^2 \Omega_\Lambda a^3)^{\frac{-1}{n}} \left[ \frac{2\sqrt{\Omega_\Lambda}}{3na} - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) \right]. \quad (20)$$

Now using Eqs. (19) and (20), the kinetic energy term and the scalar potential, Eqs. (16) and (17), can be rewritten as

$$\dot{\phi}^2 = 3M_P^2 H^2 \Omega_\Lambda \left[ \frac{2\sqrt{\Omega_\Lambda}}{3na} - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) \right], \quad (21)$$

$$V(\phi) = 3M_P^2 H^2 \Omega_\Lambda \left[ 1 - \frac{\sqrt{\Omega_\Lambda}}{3na} + \frac{b^2}{2} \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) \right]. \quad (22)$$

If we take  $\Omega_\Lambda = 0.72$ ,  $\Omega_k = 0.02$  (Bennett et al. 2003),  $n = 2.7$  (Wei and Cai 2008b) and  $a = 1$  for the present time, then Eq. (21) gives  $\dot{\phi}^2 < 0$  when  $b^2 > 0.15$ . This implies that the scalar field  $\phi$  has a phantom behavior.

Using definition  $\dot{\phi} = \phi' H$ , we can rewrite Eq. (21) in terms of derivative with respect to  $x = \ln a$  as

$$\phi' = M_P \left( 3\Omega_\Lambda \left[ \frac{2\sqrt{\Omega_\Lambda}}{3na} - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) \right] \right)^{1/2}. \quad (23)$$

Finally, the evolutionary form of the scalar field can be obtained as

$$\phi(a) - \phi(0) = M_P \int_0^{\ln a} \left( 3\Omega_\Lambda \left[ \frac{2\sqrt{\Omega_\Lambda}}{3na} - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right) \right] \right)^{1/2} dx, \quad (24)$$

where we take  $a_0 = 1$  at the present time.

## 5 Conclusions

Here we considered the interacting NADE model with CDM in the non-flat FRW universe. The ADE models proposed to explain the accelerated expansion of the universe, based on the uncertainty relation of quantum mechanics as well as the gravitational effect in general relativity (Cai 2007; Wei and Cai 2008a). We established a correspondence between the NADE density and the polytropic gas model of DE. The polytropic gas model plays a very important role in the EoS fluid description of DE in cosmology (Karami et al. 2009a). We reconstructed the potential of the interacting new agegraphic polytropic as well as the dynamics of the scalar field, which describe accelerated expansion of the universe.

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## References

- [1] Abdalla, E., Abramo, L.R., Sodre, L., Wang, B.: Phys. Lett. B **673**, 107 (2009)
- [2] Amendola, L.: Phys. Rev. D **60**, 043501 (1999)
- [3] Amendola, L.: Phys. Rev. D **62**, 043511 (2000)
- [4] Bennett, C.L., et al.: Astrophys. J. Suppl. **148**, 1 (2003)
- [5] Bertolami, O., Gil Pedro, F., Le Delliou, M.: Phys. Lett. B **654**, 165 (2007)
- [6] Bertolami, O., Gil Pedro, F., Le Delliou, M.: Gen. Rel. Grav. **41**, 2839 (2009)
- [7] Cai, R.G.: Phys. Lett. B **657**, 228 (2007)
- [8] Caldera-Cabral, G., Maartens, R., Ureña-López, L.A.: Phys. Rev. D **79**, 063518 (2009)
- [9] Cohen, A., Kaplan, D., Nelson, A.: Phys. Rev. Lett. **82**, 4971 (1999)
- [10] Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D **15**, 1753 (2006)
- [11] de Bernardis, P., et al.: Nature **404**, 955 (2000)
- [12] Feng, C., Wang, B., Gong, Y., Su, R.-K.: J. Cosmol. Astropart. Phys. **09**, 005 (2007)
- [13] Gao, C., Wu, F., Chen, X., Shen, Y.G.: Phys. Rev. D **79**, 043511 (2009)
- [14] Granda, L.N., Oliveros, A.: Phys. Lett. B **671**, 199 (2009)
- [15] Guo, Z.K., Ohta, N., Tsujikawa, S.: Phys. Rev. D **76**, 023508 (2007)
- [16] Horava, P., Minic, D.: Phys. Rev. Lett. **85**, 1610 (2000)

- [17] Huang, Q.G., Li, M.: J. Cosmol. Astropart. Phys. **08**, 013 (2004)
- [18] Hu, B., Ling, Y.: Phys. Rev. D **73**, 123510 (2006)
- [19] Karami, K., Ghaffari, S., Fehri, J.: Eur. Phys. J. C **64**, 85 (2009a)
- [20] Karami, K., Khaledian, M.S., Felegary, F., Azarmi, Z.: Preprint, arXiv:0912.1536 (2009b)
- [21] Karami, K.: J. Cosmol. Astropart. Phys. **01**, 015 (2010)
- [22] Karami, K., Fehri, J.: Phys. Lett. B **684**, 61 (2010)
- [23] Kim, H., Lee, H.W., Myung, Y.S.: Phys. Lett. B **632**, 605 (2006)
- [24] Kim, K.Y., Lee, H.W., Myung, Y.S.: Phys. Lett. B **660**, 118 (2008a)
- [25] Kim, Y.W., et al.: Mod. Phys. Lett. A **23**, 3049 (2008b)
- [26] Li, M.: Phys. Lett. B **603**, 1 (2004)
- [27] Li, M., Li, X.D., Wang, S., Wang, Y., Zhang, X.: J. Cosmol. Astropart. Phys. **12**, 014 (2009a)
- [28] Li, M., Li, X.D., Wang, S., Zhang, X.: J. Cosmol. Astropart. Phys. **06**, 036 (2009b)
- [29] Neupane, I.P.: Phys. Lett. B **673**, 111 (2009)
- [30] Pavon, D., Zimdahl, W.: Phys. Lett. B **628**, 206 (2005)
- [31] Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
- [32] Perlmutter, S., et al.: Astrophys. J. **598**, 102 (2003)
- [33] Riess, A.G., et al.: Astron. J. **116**, 1009 (1998)
- [34] Seljak, U., Slosar, A., McDonald, P.: J. Cosmol. Astropart. Phys. **10**, 014 (2006)
- [35] Sheykhi, A.: Phys. Lett. B **680**, 113 (2009a)
- [36] Sheykhi, A.: Phys. Lett. B **681**, 205 (2009b)
- [37] Sheykhi, A.: Phys. Lett. B **682**, 329 (2010a)
- [38] Sheykhi, A.: Phys. Rev. D **81**, 023525 (2010b)
- [39] Spergel, D.N.: Astrophys. J. Suppl. **148**, 175 (2003)
- [40] Spergel, D.N., et al.: Astrophys. J. Suppl. **170**, 377 (2007)
- [41] Szydlowski, M.: Phys. Lett. B **632**, 1 (2006)
- [42] Tegmark, M., et al.: Phys. Rev. D **69**, 103501 (2004)

- [43] Thomas, S.: Phys. Rev. Lett. **89**, 081301 (2002)
- [44] Tsujikawa, S., Sami, M.: Phys. Lett. B **603**, 113 (2004)
- [45] Tsujikawa, S.: Phys. Rev. D **73**, 103504 (2006)
- [46] Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B **624**, 141 (2005)
- [47] Wang, B., Lin, Ch.-Y., Abdalla, E.: Phys. Lett. B **637**, 357 (2006)
- [48] Wang, B., Zang, J., Lin, Ch.-Y., Abdalla, E., Micheletti, S.: Nucl. Phys. B **778**, 69 (2007)
- [49] Wei, H., Cai, R.G.: Phys. Lett. B **660**, 113 (2008a)
- [50] Wei, H., Cai, R.G.: Phys. Lett. B **663**, 1 (2008b)
- [51] Wei, H., Cai, R.G.: Eur. Phys. J. C **59**, 99 (2009)
- [52] Wu, J.P., Ma, D.Z., Ling, Y.: Phys. Lett. B **663**, 152 (2008)
- [53] Zhang, J., Zhang, X., Liu, H.: Phys. Lett. B **651**, 84 (2007)
- [54] Zhang, J., Zhang, X., Liu, H.: Eur. Phys. J. C **54**, 303 (2008)
- [55] Zhang, X., Wu, F.Q.: Phys. Rev. D **72**, 043524 (2005)
- [56] Zhang, X.: Phys. Rev. D **74**, 103505 (2006)
- [57] Zhang, X.: Phys. Lett. B **648**, 1 (2007)
- [58] Zhang, X., Wu, F.Q.: Phys. Rev. D **76**, 023502 (2007)
- [59] Zhang, X.: Phys. Rev. D **79**, 103509 (2009)